Using the derivative to determine if f is one-to-one

A continuous (and differentiable) function whose derivative is always positive (> 0) or always negative (< 0) is a one-to-one function. Why?

- Remember the Mean Value Theorem from Calculus 1, that says if we have a pair of numbers x_1 and x_2 which violate the condition for 1-to1ness; namely $x_1 \neq x_2$ and $f(x_1) = f(x_2)$, then there must be some point c in the interval between x_1 and x_2 with $f'(c) = 0$ (assuming that f' exists on the given interval).
- So if a function f always has a strictly positive derivative or a strictly negative derivative, we cannot find a pair of umbers x_1 and x_2 which violate the condition for 1-to1ness and the function is 1-to-1.
- This gives us a new way to check if a differentiable function f is 1-to-1. We calculate the derivative f', if we can tell that it is always positive or always negative then we can conclude that f is a 1-to-1 function.
- Note that if the function is not differentiable of its derivatives are not all strictly positive or strictly negative, we have no conclusion from this "test".

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- Example: Let $f(x) = \sqrt{4x+4} = (4x+4)^{1/2}$; is f a one-to-one function?
- Using the chain rule we have

$$f'(x) = \frac{1}{2}(4x+4)^{-1/2} \cdot 4 = \frac{2}{\sqrt{4x+4}} \quad \text{on the interval} \quad (-1, \infty)$$

- Since $f'(x) > 0$ for all x on the interval $(-1, \infty)$, we can conclude that this function is one-to-one on the interval $\{x | x > -1\}$
- The domain of f is the interval $[-1, \infty)$, however $f(-1) = 0$ which does to coincide with $f(x)$ for any x in the interval $(-1, \infty)$, so our function is one-to-one on its domain.